Fano-Rashba Effect in Quantum Wires

David Sánchez
Department of Physics
University of the Balearic Islands

In collaboration with: Llorenç Serra (UIB)
The Rashba interaction

- SO interaction in semiconductors: Rashba (SIA), Dresselhauss (BIA)
- Potential confinement \( \vec{E} || \vec{z} \)
- Rashba term important in measurements of:
  - Shubnikov - de Haas oscillations
  - Raman scattering
  - Weak localization

Rashba parameter

\[ \mathcal{H}_R = \frac{\alpha}{\hbar} (p_y \sigma_x - p_x \sigma_y) \]

- tunability with electric gates

Basis of the proposed spin-FET ➔ spintronics

Nitta et al. ‘97

No working device yet because...
(Schmidt et al, ’00)
- Velocity mismatch
- Spin relaxation
- Ferromagnetic semiconductors?
Quasi-1D system

With **only** Rashba “precession term”

\[ E_{n\pm}(k) = \varepsilon_n + \frac{\hbar^2 k^2}{2m} \pm \alpha k \quad ; \quad \psi_{nk\pm}(x, y, \eta) = \phi_n(y) e^{ikx} \chi_{y\pm}(\eta) \]

With “Rashba intersubband coupling” (RIC)

Moroz and Barnes ’99, Mireles and Kirczenow ’01, Governale and Zülicke ’02, ...
Finite Rashba region

Sánchez & Serra ´06

Detection of entangled states via shot noise. Egues, Burkard and Loss ´02

Experimental confirmation of spin interferometry with Rashba interaction ´05
The quantum transmitting boundary (QTBM) algorithm
Lent and Kirkner '89

\[ -\frac{\hbar^2}{2m} \Delta + W(y) + V(x, y; \eta \eta') \Psi(x, y; \eta') = E \Psi(x, y; \eta) \]

Grid

\[ \eta, \eta' \equiv \uparrow, \downarrow \]

in the leads \( V(x, y; \eta \eta') = 0 \rightarrow W(y) \)

- Schrödinger equation
- asymptotic equation

\( x \text{-} y \) grid: 2N unknowns
\( 'w.f. \text{ on grid points}' \)
boundary conditions
\( (\# \text{ of equations}) = (\# \text{ of grid points}) \)
Conductance

units $\ell_\alpha = \hbar^2 / m \alpha$, $E_\alpha = \hbar^2 / ml_\alpha^2$
parameters $L = 10 \ell_\alpha$, $\hbar \omega_0 = 2.5 E_\alpha$

no Rashba  no RIC  full

- boring...
- oscillations
- oscillations
- dips

Dips are due to backscattering from impurities with bound states
Faist, Guéret and Rothuizen ‘90, Gurvitz and Levinson ‘93, Nöckel and Stone ‘94
Wavefunctions

$E_a = 0.8\hbar\omega_0$

$E_b = 1.31\hbar\omega_0$

What is the origin?
A local Rashba interaction forms bound states

**NO potential other than inhomogeneous Rashba**

[In 2D: Valín-Rodríguez et al. ‘05, Cserti et al. ‘06]

Purely 1d problem

\[
\frac{1}{2} \left( \alpha(x) p_x + p_x \alpha(x) \right) \sigma_y
\]

Negative energies

\[
\frac{d^2}{dx^2} \pm 2i k_R(x) \frac{d}{dx} \psi(x) = -\kappa^2 \psi(x)
\]

\[
k_R = \alpha m / \hbar^2
\]

\[
\kappa = \sqrt{-2mE/\hbar^2}
\]

Transformation

\[
\tilde{\psi}(x) = \psi(x) e^{\pm i \int^x k_R(x) dx}
\]

\[
\tilde{\psi}''(x) + k_R(x)^2 \tilde{\psi}(x) = -\kappa^2 \tilde{\psi}(x)
\]

\[
V(x) = -\frac{\hbar^2}{2m} k_R(x)^2
\]

✅ Matching methods
✅ Lattice models
**Coupled Channel theory**

\[ -\frac{\hbar^2}{2m} \Delta + W(y) + V(x, y; \eta \eta') \]

\[ \Psi(x, y; \eta') = E \Psi(x, y; \eta) \]

**Four-mode expansion**

\[ \Psi(x, y; \eta) = \psi_1(x) \phi_1(y) \chi_+(\eta) + \psi_2(x) \phi_1(y) \chi_-(\eta) + \psi_3(x) \phi_2(y) \chi_+(\eta) + \psi_4(x) \phi_2(y) \chi_-(\eta) \]

**Projection**

\[ \sum_{\eta} \int dy \, \phi_n(y) \chi_s(\eta) \times \text{(Schrödinger Eq.)} \]

\[ \psi_1 \text{ couples with } \psi_4, \ \psi_2 \text{ with } \psi_3 \]
$$\psi_{1-\psi_{4} \text{ problem}}$$

$$\tilde{\psi}(x) = \psi(x)e^{i \int^x k_R(x)dx}$$

\[
\begin{align*}
(E - E_1) + \frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{\hbar^2}{2m} k_R(x)^2 & \quad \tilde{\psi}_1(x) = \hat{V}_{14}(x)\tilde{\psi}_4(x) \\
(E - E_2) + \frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{\hbar^2}{2m} k_R(x)^2 & \quad \tilde{\psi}_4(x) = \hat{V}_{41}(x)\tilde{\psi}_1(x)
\end{align*}
\]

$$\hat{V}_{14}(x) = -i\omega_p \frac{\alpha(x)}{\hbar} e^{-2i \int^x k_R(x)dx}$$

$$\omega_p = \langle \phi_1 | p_y | \phi_2 \rangle$$

solution by \textit{ansatz}

$$\tilde{\psi}_4(x) = A\psi_b(x)$$

bound state

$$|\tilde{\psi}_1\rangle = |\chi_k^{(1)} \rangle + \hat{G}_1 \hat{V}_{14} A |\psi_b\rangle$$

outgoing wave

retarded Green function

$$G_1(x, x') = \frac{m}{i\hbar^2 k t} \chi_k^{(1)}(x>)\chi_k^{(2)}(x<)$$
Asymptotic solution

\[
\tilde{\psi}_1(x) = \chi_k^{(1)}(x) \left[ 1 + \frac{m}{i\hbar^2 k_t} \langle \chi_k^{(2)*} | \hat{V}_{14} | \psi_b \rangle \langle \psi_b | \hat{V}_{41} | \chi_k^{(1)} \rangle}{E - \varepsilon_b - E_2 - \langle \psi_b | \hat{V}_{14} \hat{G}_1 \hat{V}_{41} | \psi_b \rangle} \right]
\]

\[\Delta + i\Gamma\]

\[T(E) = |t|^2 \frac{(E - \varepsilon_b - E_2 - \Delta + \delta)^2 + (\gamma - \Gamma)^2}{(E - \varepsilon_b - E_2 - \Delta)^2 + \Gamma^2}\]

Generalized Fano lineshape

\[T(\varepsilon) \sim \frac{|\varepsilon + q|^2}{\varepsilon^2 + 1}\]

\[q = \delta/\Gamma + i(\gamma/\Gamma - 1)\]

\[q \to \infty \text{ symmetric Breit-Wigner peak}\]

\[q \to 0 \text{ symmetric Breit-Wigner dip}\]

\[q \neq 0 \text{ asymmetric Fano lineshape}\]

Kobayashi et al. 2002
Conductance

- CCM
- Exact

Large dip width close to the 2nd plateau onset

\[ \ell_\alpha = \hbar^2 / m_\alpha = l_0, \quad L = 0.75 l_0 \]
$E_F = 1.45\hbar\omega_0, L = 8l_0$

dip minimum $\sim$ zero

renormalized quasi-bound states

Rashba interaction as a gate voltage
Spin effects

\[ T = \sum_{s,s'} T_{s,s'} = T_{\uparrow,\uparrow} + T_{\uparrow,\downarrow} + T_{\downarrow,\uparrow} + T_{\downarrow,\downarrow} \]
Conclusions

- Quantum wires with local Rashba interaction
- Conductance dips before plateau onset
- Interference between direct transmission and bound states \( \rightarrow \) Fano-Rashba effect
- Both resonance position and broadening depend on \( \alpha \)
- Strong spin effects