VIIIth Rencontres du Vietnam - Beyond the Standard Model

Generation of Large-Scale Magnetic Fields from Inflation in Teleparallelism

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Outline

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Riemann-Cartan Geometry U_4

- Gravitational theory described on Riemannian geometry V_4 with metric $g_{\mu\nu}$ and metric-compatible Levi-Civita connection

$$\{{}^{\rho}_{\mu\nu}\} = -\frac{1}{2}g^{\rho\lambda}(g_{\nu\lambda,\mu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda})$$

is Einstein's general relativity established in 1915.

- The metric-compatible affine connection in U_4 is

$$\Gamma^{\rho}_{\mu\nu} = \{{}^{\rho}_{\mu\nu}\} + K^{\rho}{}_{\mu\nu},$$

where the second term is contorsion tensor

$$K^{\rho}{}_{\mu\nu} = \frac{-1}{2}(T^{\rho}{}_{\mu\nu} - T_{\mu}{}^{\rho}{}_{\nu} - T_{\nu}{}^{\rho}{}_{\mu})$$

with the torsion tensor is the antisymmetric part of affine connection

$$T^{\rho}{}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu}.$$

- Introducing torsion: in order to couple spin angular momentum in gravity (Élie Cartan 1922.)

- Gravity as a gauge theory

Einstein-Cartan-Sciama-Kibble theory (ECSK)

$$S_{ECSK} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R(g,T) \right].$$

- Include massless Rarita-Schwinger field (gravitino, Rarita & Schwinger 1941)
  $\rightarrow$ Supergravity (gauge SUSY)

- Include torsion part in the Lagrangian
  $\rightarrow$ Poincaré gauge theory

What is Teleparallelism?

- A special case of Poincaré gauge theory (only quadratic torsion part) with absolute parallelism.
- Introduce the orthonormal frame (veirbein)

$$g_{\mu\nu} = \eta_{AB} e_\mu^A e_\nu^B = e_{\mu B} e_\nu^B \to \sqrt{-g} = e.$$

- Weitzenböck geometry W_4 is define by the metric-compatible Weitzenböck connection $W^\rho{}_{\mu\nu} = e_A^\rho \partial_\mu e_\nu^A$ with Riemann tensor $R^\sigma{}_{\rho\mu\nu}(W) = 0$.
- The gravitaty theory on Weitzenböck geometry is so-called new general relativity (NGR) (Hayashi & Shirafuji 1979.)
- The torsion tensor in terms of the vierbein is

$$T^\rho{}_{\mu\nu} \equiv e_A^\rho \left(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A\right).$$

NGR Lagrangian

$$\mathcal{L}_{NGR} = \frac{1}{2\kappa^2}\, e\left[a_1\; T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + a_2\; T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} + a_3\; T^{\rho}{}_{\mu\rho}T^{\nu\mu}{}_{\nu}\right].$$

- The vector part of torsion tensor is $v_\mu \equiv T^{\nu}{}_{\nu\mu}$ and the axial-vector part is $a_\mu \equiv \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma}$.
- The NGR Lagrangian can be rewritten to

$$\mathcal{L}_{NGR} = \frac{1}{2\kappa^2} e\left[b_1\; t^{\rho}{}_{\mu\nu}t_{\rho}{}^{\mu\nu} + b_2\; v^{\mu}v_{\mu} + b_3\; a^{\mu}a_{\mu}\right].$$

- Teleparallel Equivalent to GR:

$$R(W) = R(\{\}) + T - 2\underbrace{\nabla_\mu v^\mu}_{\text{boundary term}} \equiv 0.$$

TEGR Lagrangian

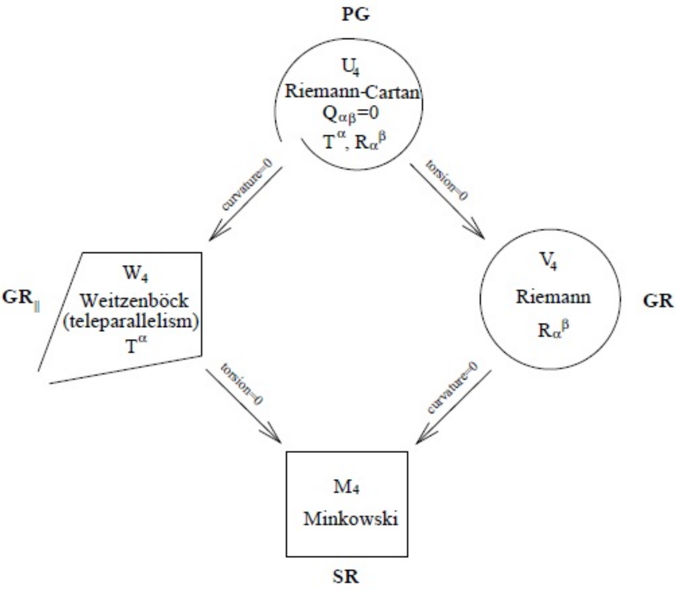
$$\mathcal{L}_{TEGR} = \frac{1}{2\kappa^2}\, e\left[\frac{1}{4}T^\rho{}_{\mu\nu}T_\rho{}^{\mu\nu} + \frac{1}{2}T^\rho{}_{\mu\nu}T^{\nu\mu}{}_\rho - T^\rho{}_{\mu\rho}T^{\nu\mu}{}_\nu\right].$$

- Sometimes written the form

$$\mathcal{L}_{TEGR} \equiv \frac{1}{2\kappa^2} e\, T$$

and $T \equiv S_\rho{}^{\mu\nu}T^\rho{}_{\mu\nu}$ with $S_\rho{}^{\mu\nu} \equiv \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho\, T^{\sigma\nu}{}_\sigma - \delta^\nu_\rho\, T^{\sigma\mu}{}_\sigma\right).$

[arXiv:gr-qc/9602013]



Outline

Mystery

The origin of the magnetic fields observed in the galaxies and galaxy clusters.

- Magnectic fields with strength $10^{-6}\mathrm{G}$ at coherence scale $1-10\mathrm{kpc}$.
- $10^{-7}-10^{-6}\mathrm{G}$ Large-scale magnetic fields at coherence scale $10\mathrm{kpc}-1\mathrm{Mpc}$.

- Future experiments on CMB radiation
  - PLANCK
  - QUIET
  - B-Pol
  - LiteBIRD

- Dynamo mechanism: the kinetic energy associated with the differential rotation is converted into magnetic field energy.
- Galactic dynamo: to amplify the energy of seed magnetic field of spiral galaxies into the $10^{-6}\mathrm{G}$ strength fields observed in the galaxies.
- Some generation mechanisms (not natural)
  - Plasma instability
    $\rightarrow$ Magnetohydrodynamics (MHD): the interaction between magnectic fields and plasmas.
  - Cosmological phase transition
    $\rightarrow$ Magnetogensis: need out-of-thermal equilibrium and macroscopic parity violation.
  - Matter perturbation before or after the recombination

The most natural mechanism

EM quantum fluctuation during inflation (Turner & Widrow 1988)
$\rightarrow$ Break conformal symmetry by non-minimal coupling.

- From quantum electrodynamics in curved space-time the non-minimal coupling is owing to the one-loop vacuum-polarization effect (Drummond & Hathrell 1980.)

Outline

Model

- From Einstein equation
  $\rightarrow$ two classes to explain the accelerating expansion.

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{modified gravity}} = \underbrace{8\pi G T_{\mu\nu}}_{\text{modified matter}} .$$

- The action of inflation model in teleparallelism is

$$S = \int d^4x \, e \left[\frac{f(T)}{2\kappa^2} + \mathcal{L}_{\mathrm{M}} \right] .$$ (Ferraro & Fiorini 2007)

- The electromagnetic sector is described by a non-minimal $I(T)$-Maxwell theory

$$S_{em} = \int d^4x\, e \left(-\frac{1}{4} I(T)\, F_{\mu\nu} F^{\mu\nu} \right),$$

with second quantization

$$\begin{aligned} A_i(t, \boldsymbol{x}) &= \int d^3k (2\pi)^{-3/2} \sum_{\sigma=1,2} [\, \hat{b}(\boldsymbol{k}, \sigma) \epsilon_i(\boldsymbol{k}, \sigma) A(t, k) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}} \\ &\quad + \hat{b}^\dagger(\boldsymbol{k}, \sigma) \epsilon_i^*(\boldsymbol{k}, \sigma) A^*(t, k) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \,] \end{aligned}$$

- The EoM by using conformal time η is

$$A''(k, \eta) + (I'/I)\, A'(k, \eta) + k^2 A(k, \eta) = 0.$$

Analytical Solution for Magnetic Field

- By solving the EoM lead to $|A(k,\eta)|^2$ at the late times with

$$A(k) \equiv \frac{1}{\sqrt{2k}} I^{-1/2} \left[1 - \left(\frac{1}{2} I' + ikI\right) \int_{\eta}^{\eta_{\rm R}} \frac{1}{I\left(\tilde{\tilde{\eta}}\right)} d\tilde{\tilde{\eta}}\right] \mathrm{e}^{-ik\eta}\Big|_{\eta=\eta_k}$$

- By using the comoving magnetic field $B_i(t,\boldsymbol{x})$, the proper magnetic field is expressed as

$$B_i{}^{\rm proper}(t,\boldsymbol{x}) = a^{-1} B_i(t,\boldsymbol{x}) = a^{-2} \epsilon_{ijk} \partial_j A_k(t,\boldsymbol{x}).$$

- The energy density of the generated magnetic field per unit logarithmic interval of k is

$$\rho_B(k,\eta) \equiv \frac{1}{2} \frac{4\pi k^3}{(2\pi)^3} |B^{\rm proper}(k,\eta)|^2 I(\eta) = \frac{k|A(k)|^2}{2\pi^2} \frac{k^4}{a^4} I(\eta)\,.$$

- the density parameter and spectral index are given by

$$\Omega_B(k,\eta) = \frac{\rho_B(k,\eta_{\rm R})}{\rho_\gamma(\eta_{\rm R})}\frac{I(\eta)}{I(\eta_{\rm R})} = \frac{k^4}{T_{\rm R}^4 a_{\rm R}^4}\frac{15k|A(k)|^2}{N_{\rm eff}\pi^4}I(\eta)\,,$$

$$n_B \equiv \frac{d\ln\Omega_B(k)}{d\ln k} = 4 + \frac{d\ln k|A(k)|^2}{d\ln k}\,,$$

where $\rho_\gamma(\eta_{\rm R}) = N_{\rm eff}\left(\pi^2/30\right)T_{\rm R}^4$.

- Set the specific form of non-minimal coupling

$$I(\eta) = I_*\left(\frac{\eta}{\eta_*}\right)^{-\beta}\,,$$

with $I_* = I(\eta_*)$ and we obtain

$$k|A_1|^2 = \frac{1}{2I(\eta_k)}\left|\frac{1-(\beta+2i)}{2(\beta+1)}\right|^2 \equiv \frac{\mathcal{A}}{2I(\eta_k)},$$

where $\mathcal{A}(=\mathcal{O}(1))$ is a constant of the order of unity.

- From the relation $|B(\eta_0, k)|^2 = 2\rho_B(\eta_0, k) = 2\Omega_B(\eta_0, k)\,\rho_\gamma(\eta_0)$ with $\rho_\gamma(\eta_0) \simeq 2 \times 10^{-51}\,\mathrm{GeV}^4$ and $1\,\mathrm{G} = 1.95 \times 10^{-20}\,\mathrm{GeV}^2$

The current amplitude of the magnetic field

$$|B(\eta_0, L)| = 2.7 \left[\frac{7.2}{(5.1)^4 \pi}\right]^{\beta/8} \times 10^{-56+51\beta/4} N_{\mathrm{eff}}^{(\beta-4)/8} \times \sqrt{\mathcal{A} \frac{I(\eta_0)}{I(\eta_{\mathrm{R}})}} \left(\frac{H_{\mathrm{R}}}{M_{\mathrm{Pl}}}\right)^{\beta/4} \left(\frac{L}{[\mathrm{Mpc}]}\right)^{\beta/2-2} \mathrm{G}\,.$$

Numerical Results

- Suppose the power-law inflation ($a = a_0(t/t_0)^p$), the relation between t and η is

$$\frac{t}{t_0} = \left[a_0 t_0 \left(p-1\right)\left(-\eta\right)\right]^{-1/(p-1)} .$$

- The case of a power-law type coupling $I(T) = (T/T_0)^n$

$$I(\eta) = \left(-6/T_0\right)^n \left(p/t_0\right)^{2n} \left[a_0 t_0 \left(p-1\right)\right]^{2n/(p-1)} \left(-\eta\right)^{2n/(p-1)} .$$

- $N_{\rm eff} = 100$, $H_{\rm R} = 1.0 \times 10^{14} GeV$ ($T_{\rm R} = 8.6 \times 10^{15} GeV$), $L = 1$Mpc, $\mathcal{A} = 1$, $I(\eta_{\rm R}) = I(\eta_0)$, and $\beta = 4.2$

$$|B(\eta_0, L = 1\,\mathrm{Mpc})| = 2.5 \times 10^{-9}\,\mathrm{G} .$$

- Similarly, $H_{\rm R} = 1.0 \times 10^{10} GeV$ ($T_{\rm R} = 8.6 \times 10^{13} GeV$) and $\beta = 4.6$, met for $p = 10$ and $n = -19.7$

$$|B(\eta_0, L = 1\,\mathrm{Mpc})| = 2.3 \times 10^{-9}\,\mathrm{G} .$$

Outline

Summary and Conclusions

- Current observation of the large-scale magectic field strength is $10^{-6}\mathrm{G}$.
- Large-scale magectic field can be naturally generated through the conformal symmetry broken by non-minimal coupling of torsion and electromagnetic fields in the teleparallesim during inflation.
- The magnetic field with its strength of $\sim 10^{-9}\mathrm{G}$ and the coherence scale of 1Mpc at the present time can be generated.

Resultant field strength is compatible with the upper limit of $\sim 2\text{–}6 \times 10^{-9}\mathrm{G}$ obtained from CMB radiation as well as that of being smaller than $4.8 \times 10^{-9}\mathrm{G}$ from CMB radiation on the present strength with scales larger than the present horizon.

The constraint on the current strength of the magnetic fields on the BBN horizon scale $\sim 9.8 \times 10^{-5} h^{-1}\mathrm{Mpc}$ $(h = 0.7)$, is smaller than $10^{-6}\mathrm{G}$.

End

- Thank you!

Outline

Observational Methods

- Synchrotron emission: from pulsar to superclusters.
- Faraday rotation: the change of the polarization angle.
- Zeeman splitting (energy splitting): eg. 21-cm hydrogen line.
- Polarization of optical starlight: can reveal the presence of large-scale magnetic fields in our Galaxy and nearby.